

## RAMA DISTRIBUTED CASCADE MODEL WITH POISSON PROCESS

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### Abstract

*In the present paper time dependent  $n$ -cascade systems for  $n = 1, 2, 3, 4$  have been considered where the number of impacts of stresses faced by the system is a Poisson process. For the estimation of cascade systems reliability  $R_n$ , it has been considered that both stress and strength follow Rama distribution. The expressions for the system reliability of the time dependent cascade systems, the mean and variance are given for single component. The numerical values of  $R_1(t)$  and  $R_2(t)$  have also been computed and provided in tabular forms for some specific values of the parameters. A simple case study is presented to illustrate the application of reliability concepts in structural engineering. Lastly, we have used graphs to gain a clear picture of the system reliability.*

**Keywords:** Cascade system, Stress-Strength, Rama distribution, Poisson process, Reliability

**JEL Classification:** C4

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### Introduction

Cascade systems were first developed and studied by Pandit and Sriwastav (Pandit and Sriwastav, 1975). In the interference theory of reliability, a system with certain strength works under impact of stresses. Many authors (Frudenthal, 1966), (Pandit and Sriwastav, 1975), (Kapur and Lamberson, 1977), to name a few have studied interference models in reliability without taking time into consideration. They have studied single impact systems. In such models, reliability of the system depends only on the stress- strength of the system; passage of time has no effect in it. But time is an important factor to be considered for the study of reliability. In some studies (Sriwastav, 1994) time has come into the picture in an indirect way.

To bring time into the model directly, it is assumed that stresses impinging on the system arrive as a Poisson process with parameter  $\beta$ . Reliability can now be defined as the probability that the system working under impact of stresses will survive up to time  $t$ , when the stresses impinging on the system arrive as a Poisson process with intensity  $\beta$ . Here we have considered 2-cascade, 3-cascade and 4-cascade systems. Here system reliability at time ' $t$ ' is defined as the probability that the system stands ' $r$ ' impacts i.e., at least one component is working at time ' $t$ '.

This paper is divided into six sections. The first section of the paper is introductory in nature. In Section 2 the cascade model is presented mathematically where the number of impacts of stresses faced by the system is a Poisson process. In Section 3 the expressions for the reliability, mean and variance of time-dependent single component cascade system are obtained. In section 4 the numerical values of the reliability expressions are obtained. Graphical representations of the system reliability expressions are provided in section 5. The practical implications and applications of reliability across various fields are discussed in Section 6. A simple case study illustrating the application of reliability in Structural engineering (Bridge design) is presented in Section 7 and the conclusion of the paper is provided in Section 8.

## Methodology

- Cascade Model

Let  $X_1, X_2, \dots, X_n$  be the strengths of  $n$ -components in the order of activation and let  $Y_1, Y_2, \dots, Y_n$  are the stresses working on them. In Cascade system (Doloi et al., 2010) after every failure the stress is modified by a factor  $k$  such that

$$Y_2 = kY_1, Y_3 = kY_2 = k^2Y_1, \dots, Y_i = k^{i-1}Y_1 \text{ etc.}$$

where,  $Y_1$  is the stress on the first component.

Now the marginal reliability  $R(1), R(2), R(3)$  and  $R(4)$  may be obtained as

$$R(1) = \int_{-\infty}^{\infty} \bar{F}(y_1)g(y_1) dy_1 \quad (1)$$

$$R(2) = \int_{-\infty}^{\infty} F(y_1)\bar{F}(ky_1)g(y_1) dy_1 \quad (2)$$

$$R(3) = \int_{-\infty}^{\infty} F(y_1)F(ky_1)\bar{F}(k^2y_1)g(y_1) dy_1 \quad (3)$$

$$R(4) = \int_{-\infty}^{\infty} F(y_1)F(ky_1)F(k^2y_1)\bar{F}(k^3y_1)g(y_1) dy_1 \quad (4)$$

The reliability of an  $n$ -cascade system (Pandit and Sriwastav, 1975) is given as

$$R_n = R(1) + R(2) + \dots + R(n) \quad (5)$$

where,  $r^{\text{th}}$  component marginal reliability may be given as

$$R(r) = P[X_1 < Y_1, X_2 < kY_1, \dots, X_{r-1} < k^{r-2}Y_1, X_r \geq k^{r-1}Y_1] \quad (6)$$

The number  $r$  of impacts of stresses on the system during time  $(0, t)$  following a Poisson distribution is given by (Gogoi et al., 2010)

$$p(r) = \frac{e^{-\beta t} (\beta t)^r}{r!}, \quad r = 0, 1, 2, \quad (7)$$

In an  $n$ -standby system, initially there are  $n$  components out of which only one is working under impact of stresses and the remaining  $(n-1)$  are cold standbys. Here we would like to note that atleast one impact is required for the failure of a component and more than one component may fail in a single impact. We have to obtain first the probability that the system survives ' $r$ ' shocks. Here we have considered the case for  $n \leq 4$ , the expressions for higher values of  $n$  becomes cumbersome.

If  $R_n(r)$ ,  $n = 1, 2, 3, 4$  gives the reliability of an  $n$ -cascade system at the  $r^{\text{th}}$  attack then following (Sriwastav, 2003) we have

$$R_1(r) = R_1^r \tag{8}$$

$$R_2(r) = R_1^r + R(2) \binom{r}{1} R_1^{r-1} \tag{9}$$

$$R_3(r) = R_2(r) + R(3) \binom{r}{1} R_1^{r-1} + \{R(2)\}^2 \binom{r}{2} R_1^{r-2} \tag{10}$$

$$R_4(r) = R_3(r) + R(4) \binom{r}{1} R_1^{r-1} + 2 R(2)R(3) \binom{r}{2} R_1^{r-2} + \{R(2)\}^3 \binom{r}{3} R_1^{r-3} \tag{11}$$

where,  $R(1) = R_1$

Since the random variable  $r$  follows a Poisson distribution given in (7), reliability  $R_n(t)$  at time  $t$  is given by

$$R_n(t) = \sum_{r=0}^{\infty} p(r) R_n(r) \tag{12}$$

Substituting the values of  $p(r)$  from (2.7) and  $R_n(r)$  from (8) to (11), we get

$$R_1(t) = e^{-\beta t(1-R_1)} \tag{13}$$

$$R_2(t) = R_1(t) [1 + \beta t R(2)] \tag{14}$$

$$R_3(t) = R_1(t) \left[ 1 + \beta t \{R(2) + R(3)\} + \frac{(\beta t)^2 \{R(2)\}^2}{2!} \right] \tag{15}$$

$$R_4(t) = R_1(t) \left[ 1 + \beta t \{R(2) + R(3) + R(4)\} + \frac{(\beta t)^2 R(2) \{R(2) + 2R(3)\}}{2!} + \frac{(\beta t)^3 \{R(2)\}^3}{3!} \right] \tag{16}$$

For some specific stress-strength distributions we can obtain  $R(i)$ ,  $i = 1, 2, 3, 4$  from (6) and substituting these results in (13) to (16) we can get  $R_n(t)$  for  $n \leq 4$ .

### Results and Discussions

- Stress-Strength follow Rama distributions

Let  $X$  and  $Y$  are independent strength and stress random variables having Rama distribution (Shanker, 2017) with parameter  $\theta_1$  and  $\theta_2$ , respectively. Then, the stress-strength reliability  $R$  of Rama distribution can be obtained as

$$R = P(Y < X) = \int_0^{\infty} P(Y < X / X = x) f_x(x) dx$$

$$= \int_0^{\infty} f(x; \theta_1) F(x; \theta_2) dx$$



$$A = \frac{1}{(\theta^3 + 6)^3} \left[ \begin{aligned} & \frac{1}{\theta + \theta k} \{ \theta^{10} + 12\theta^7 + 36\theta^4 \} + \frac{1!}{(\theta + \theta k)^2} \{ 6k\theta^8 + 36k\theta^5 \} + \frac{2!}{(\theta + \theta k)^3} \{ 3k^2\theta^9 + 18k^2\theta^6 \} \\ & + \frac{3!}{(\theta + \theta k)^4} \{ \theta^{10} + k^3\theta^{10} + 12\theta^7 + 18k^3\theta^7 + 72\theta^4 \} + \frac{4!}{(\theta + \theta k)^5} \{ 6k\theta^8 + 36k\theta^5 \} + \\ & \frac{5!}{(\theta + \theta k)^6} \{ 3k^2\theta^9 + 18k^2\theta^6 \} + \frac{6!}{(\theta + \theta k)^7} \{ k^3\theta^{10} + 6k^3\theta^7 \} - \frac{1}{2\theta + \theta k} \left\{ \frac{\theta^{10} + 12\theta^7 +}{36\theta^4} \right\} \\ & - \frac{1!}{(2\theta + \theta k)^2} \{ 6\theta^8 + 6k\theta^8 + 36k\theta^5 + 36\theta^5 \} - \frac{2!}{(2\theta + \theta k)^3} \left\{ \frac{6\theta^9 + 3k^2\theta^9 + 18\theta^6 +}{36k\theta^6 + 18k^2\theta^6} \right\} \\ & - \frac{3!}{(2\theta + \theta k)^4} \{ 2\theta^{10} + k^3\theta^{10} + 18\theta^7 + 18k^3\theta^7 + 18k\theta^7 + 18k^2\theta^7 + 72\theta^4 \} - \frac{4!}{(2\theta + \theta k)^5} \\ & \left\{ \frac{12k\theta^8 + 9k^2\theta^8 + 6k^3\theta^8 + 36k\theta^5 + 36k\theta^5}{(2\theta + \theta k)^6} \right\} - \frac{5!}{(2\theta + \theta k)^6} \left\{ \frac{3\theta^9 + 6k^2\theta^9 + k^3\theta^9 +}{18k^2\theta^6 + 36k\theta^6 + 18\theta^6} \right\} \\ & - \frac{6!}{(2\theta + \theta k)^7} \{ \theta^{10} + 2k^3\theta^{10} + 6k^3\theta^7 + 18k^2\theta^7 + 18k\theta^7 + 6\theta^7 \} - \\ & \frac{7!}{(2\theta + \theta k)^8} \left\{ \frac{6k^3\theta^8 +}{9k^2\theta^8 + 6k\theta^8} \right\} - \frac{8!}{(2\theta + \theta k)^9} \{ 3k^3\theta^9 + 3k^2\theta^9 \} - \frac{9!}{(\theta + \theta k)^{10}} \{ k^3\theta^{10} \} \end{aligned} \right]$$

For this distribution we find that the moment generating function for single component cascade system i.e.,

$$M_1(\alpha) = \frac{\beta}{\beta \left[ 1 - \frac{1}{(\theta^3 + 6)^2} \left\{ \frac{\theta^6}{2} + \frac{41}{8}\theta^3 + \frac{79}{128} \right\} \right]} - \alpha \left[ 1 - \frac{1}{(\theta^3 + 6)^2} \left\{ \frac{\theta^6}{2} + \frac{41}{8}\theta^3 + \frac{79}{128} \right\} \right]$$

Hence,

$$\begin{aligned} \text{Mean } \mu_1' &= \frac{-1}{\beta \left[ 1 - \frac{1}{(\theta^3 + 6)^2} \left\{ \frac{\theta^6}{2} + \frac{41}{8}\theta^3 + \frac{79}{128} \right\} \right]} \\ \text{Variance } \mu_2 &= \frac{1}{\beta^2 \left[ 1 - \frac{1}{(\theta^3 + 6)^2} \left\{ \frac{\theta^6}{2} + \frac{41}{8}\theta^3 + \frac{79}{128} \right\} \right]^2} \end{aligned}$$

### Numerical Evaluation

For some specific values of the parameters involved in the expressions of  $R_n(t)$ ,  $n=1,2$  we evaluate  $R_1(t)$  and  $R_2(t)$  for different values of  $\beta$ ,  $t$  and  $\theta$  from their expressions obtained in the last section.

**Table 1: Both stress and strength are follows Rama distribution for k =.5**

$\beta$	$t$	$\theta$	$R_1(t)$	$R_2(t)$
1	1	3	0.9213	0.9728
		4	0.9101	0.9531
		5	0.8565	0.9414
2	2	3	0.7204	0.8454
		4	0.6225	0.7230
		5	0.5671	0.6143
3	3	3	0.4781	0.7547
		4	0.3182	0.6347
		5	0.2503	0.5127

*Source: Authors' Self-construct*

From the tabulated values we observe that, when  $\theta$  increases  $R_1(t)$  and  $R_2(t)$  decreases for fixed values of  $\beta$  and  $t$ . For example, when  $\beta = 1, t = 1, \theta = 3$  then system reliability  $R_1(t) = 0.9213$  and  $R_2(t) = 0.9728$  and when  $\beta = 1, t = 1, \theta = 4$  then system reliability  $R_1(t) = 0.9101$  and  $R_2(t) = 0.9531$ . Similarly, when  $\beta = 2, t = 2, \theta = 3$  then system reliability  $R_1(t) = 0.7204$  and  $R_2(t) = 0.8454$  and when  $\beta = 2, t = 2, \theta = 4$  then system reliability  $R_1(t) = 0.6225$  and  $R_2(t) = 0.7230$ . By proper choice of the different parameters, very high system reliability can be achieved.

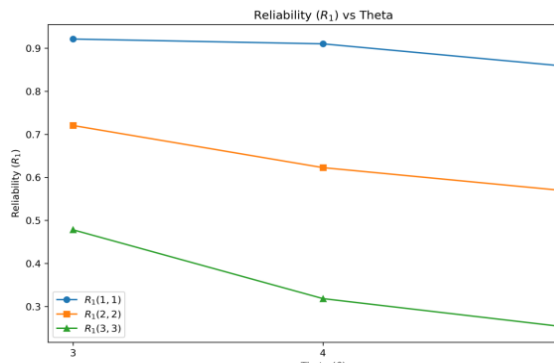
### Graphical Representation

Graphs of  $R_1(t)$  and  $R_2(t)$  are drawn in Figure 1 and Figure 2 for different parametric values involved. Taking  $\theta$  along the horizontal axis and the corresponding  $R_1(t)$  and  $R_2(t)$  along the vertical axis graphs are plotted for different pairs of  $\beta$  and  $t$ .

Figure 1 represents the curves for  $R_1(t)$  where  $\beta = 1, t = 1, \theta = 3, 4, 5$ ;  $\beta = 2, t = 2, \theta = 3, 4, 5$  and  $\beta = 3, t = 3, \theta = 3, 4, 5$ .

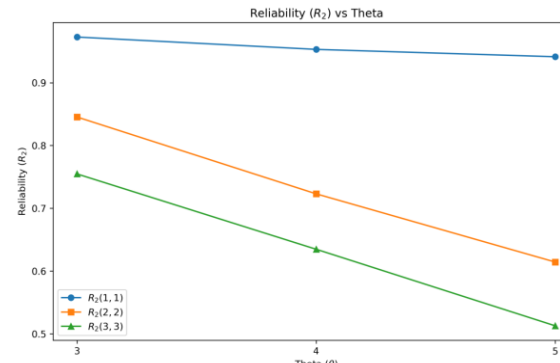
Again, Figure 2 represents the curves for  $R_2(t)$  where  $\beta = 1, t = 1, \theta = 3, 4, 5$ ;  $\beta = 2, t = 2, \theta = 3, 4, 5$  and  $\beta = 3, t = 3, \theta = 3, 4, 5$ .

**Figure 1: System Reliability  $R_1(t)$  vs  $\theta$**



*Source: Authors' Self-construct*

**Figure 2: System Reliability  $R_2(t)$  vs  $\theta$**



*Source: Authors' Self-construct*

## Practical Implications and Applications

### Reliability Engineering

**Application:** Ensures the long-term reliability of critical infrastructure systems, such as bridges, dams, or high-speed rail networks, which are subject to stresses like environmental forces, wear and tear and maintenance delays. For power grids, it forecasts blackout risks; in transportation, it evaluates subsystem reliability; and for structures, it supports real-time monitoring and resilience strategies against environmental stresses and delayed repairs, ensuring long-term system integrity.

**Implication:** By modeling the system's reliability with a time-dependent cascade model, engineers can predict time-dependent failure probabilities in critical infrastructure, enabling proactive maintenance and upgrades. This approach minimizes downtime, enhances safety, and optimizes resource allocation for sustained system reliability.

### Manufacturing and Quality Control

**Application:** In a factory setting, production lines often undergo intermittent stresses (e.g., machinery breakdowns, material inconsistencies, or unexpected environmental factors). By considering and modifying both the strength of materials and the stochastic nature of stress events, manufacturers can improve product quality, predict failure rates, and optimize maintenance schedules to minimize downtime and enhance operational efficiency.

**Implication:** The RAMA Distributed Cascade Model with Poisson Process can effectively model the stress-strength interaction for product components in a factory setting. Engineers can predict when parts are likely to fail under intermittent stresses, enabling the design of more robust components and optimized manufacturing schedules, reducing waste and costs. Additionally, this approach supports proactive maintenance strategies, enhancing production line reliability and minimizing unexpected downtime.

### Finance and Risk Management

**Application:** In financial systems and insurance markets, the RAMA Distributed Cascade Model with Poisson Process models economic shocks (e.g., market crashes, credit defaults) as Poisson-distributed events, with stress reflecting shock severity and strength representing the system's resilience. This enables risk assessment, predicts cascading failures, and optimizes strategies to enhance financial stability.

**Implication:** This model enables banks and insurance companies to better assess their exposure to significant losses over time, helping them allocate appropriate reserves and implement effective risk mitigation strategies.

### *Structural Engineering and Building Safety*

**Application:** In the construction of tall buildings or skyscrapers, a time-dependent cascade system can model how the structure will respond to external forces such as earthquakes, wind, or gradual material degradation.

**Implication:** By analyzing stress and strength distributions over time, engineers can design structures with improved safety features, ensuring that buildings remain stable even under the cumulative effects of multiple smaller stresses.

## **Case Study: Application in Structural Engineering (Bridge Design)**

### *Scenario*

Consider a civil engineering company tasked with designing a bridge that will be subjected to varying levels of stress over its lifetime. These stresses may include traffic loads, seismic activity, temperature fluctuations, and wind forces, all of which can occur randomly and over time. The company uses the time-dependent cascade system model where the following assumptions are made:

The occurrence of stress events (traffic, wind, etc.) follows a Poisson process. Both the strength of the bridge (material durability) and the stress events follow the Rama distribution.

### Steps in the Case Study

#### *Modeling Stress and Strength*

**Stress:** The intensity of stress (such as traffic, weather conditions, or external loads) can be modeled using a Poisson process. For instance, traffic intensity may fluctuate over time—showing higher rates during peak hours and significantly lower levels during off-peak periods.

**Strength:** The material strength (or durability) of the bridge is represented using the Rama distribution, which captures variations in material properties over time due to factors such as wear, aging, and fatigue.

#### *Reliability Estimation*

The reliability function for the bridge can be derived based on the time-dependent cascade model. The system's reliability  $R(t)$  is the probability that the system has not failed at time  $t$ . This function incorporates both the stress and the strength distribution, accounting for the cumulative effects of past stresses on the structure's present strength.

#### *Failure Prediction*

Over time, as stresses accumulate from factors such as traffic, wind and environmental conditions, the probability of failure increases. By combining the cumulative distribution of stress events with the Rama distribution for material strength, engineers can estimate the expected time to failure and determine optimal intervals for maintenance or reinforcement.

### *Maintenance Scheduling*

Based on the reliability estimation, the company schedules preventive maintenance. For example, if the probability of failure exceeds a critical threshold, the company might plan for reinforcement, repairs, or inspections.

### *Results*

- The time-dependent cascade system model provides the engineers with a reliable estimate of when the bridge will likely fail based on expected stress events.
- This enables the company to perform cost-effective maintenance by minimizing the risk of catastrophic failure and the total maintenance cost throughout the bridge's lifetime.

### *Summary*

In this case study, the time-dependent cascade system model—incorporating Poisson-distributed stress events and the Rama distribution for strength—enables the engineering team to accurately predict the bridge's failure likelihood and optimize its maintenance schedule. Such models are invaluable in sectors where reliability and failure prediction are critical, including infrastructure design, manufacturing, finance etc.

By applying these concepts in a real-world scenario, businesses can proactively and efficiently manage risks, improve system designs, and optimize resources, all while maintaining safety and reducing costs.

### **Conclusion**

In this paper, we propose Rama distribution for both stress-strength to obtain Cascade system reliability where the number of impacts of stresses faced by the system is a Poisson process. As a result, for this distribution, moment generating function for single component cascade system has been obtained and also mean and variance are given for single component. Numerical values of the reliability expressions are obtained and it is found that the reliabilities lie between 0 to 1, which validates the reliability expressions. Also, some graphs are added to see the clear picture of the reliability. As the proposed distribution is the new probability distribution, a lot of works can be done in the future in reliability theory.

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